

Module 2: Properties of Exponents

Topic 3: Zero and Negative Exponents

Introduction



Today's Lesson

- You will apply your knowledge of exponents.
- You will learn how to raise a value to the power of zero, or to a negative power.

Hi there! In your earlier math studies, you have had practice working with exponents. More likely than not, those exponents were usually positive whole numbers. In this lesson you will apply your knowledge of exponents to discover a rule that allows you to raise a value to the power of zero, or to a negative power.

Module 2: Properties of Exponents

Topic 3: Zero and Negative Exponents

Table of Exponents

2^4	16
2^3	8
2^2	4
2^1	2
2^0	
2^{-1}	
2^{-2}	

In order to understand how zero and negative exponents work, take a few moments to work through this table.

What is the value of 2^4 ? You could use the calculator or mental math to determine that the value is 16.

2^3 ...that's 8.

2^2 ...that's 4.

And, 2^1 ...that's simply 2.

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Table of Exponents (continued)

2^4	16
2^3	8
2^2	4
2^1	2
2^0	
2^{-1}	
2^{-2}	

Now take a moment to analyze the values in the table. Do you notice any patterns? You can determine a value in the right column of the table, by dividing the previous value by 2.

For example, the first value in the right column is 16.

$\frac{16}{2} = 8$, which is the second value in the right column.

$\frac{8}{2} = 4$, which is the third value in the right column.

And, $\frac{4}{2} = 2$, which is the fourth value in the right column.

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Table of Exponents (continued)

2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2}$
2^{-2}	$\frac{1}{4}$

If you use this pattern to complete the right column, you'll find that...

$$2^0 = 1;$$

$$2^{-1} = \frac{1}{2}; \text{ and}$$

$$2^{-2} = \frac{1}{4}.$$

The completed table shows the patterns that arise when you raise a number to the power of zero and when you raise a number to a negative power.

Module 2: Properties of Exponents

Topic 3: Zero and Negative Exponents

Definition of a Zero Exponent

2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2^1}$
2^{-2}	$\frac{1}{2^2}$

Definition of a Zero Exponent

If $a \neq 0$,
then $a^0 = 1$

In the table you determined that $2^0 = 1$.

The definition of a zero exponent states that if $a \neq 0$, then $a^0 = 1$.

In the table you also determined that... $2^{-1} = \frac{1}{2}$, which is equivalent to $\frac{1}{2^1}$; and $2^{-2} = \frac{1}{4}$ which is equivalent to $\frac{1}{2^2}$.

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Topic 3: Zero and Negative Exponents

Definition of a Negative Exponent

2^4	16
2^3	8
2^2	4
2^1	2
2^0	1
2^{-1}	$\frac{1}{2^1}$
2^{-2}	$\frac{1}{2^2}$

Definition of a Negative Exponent

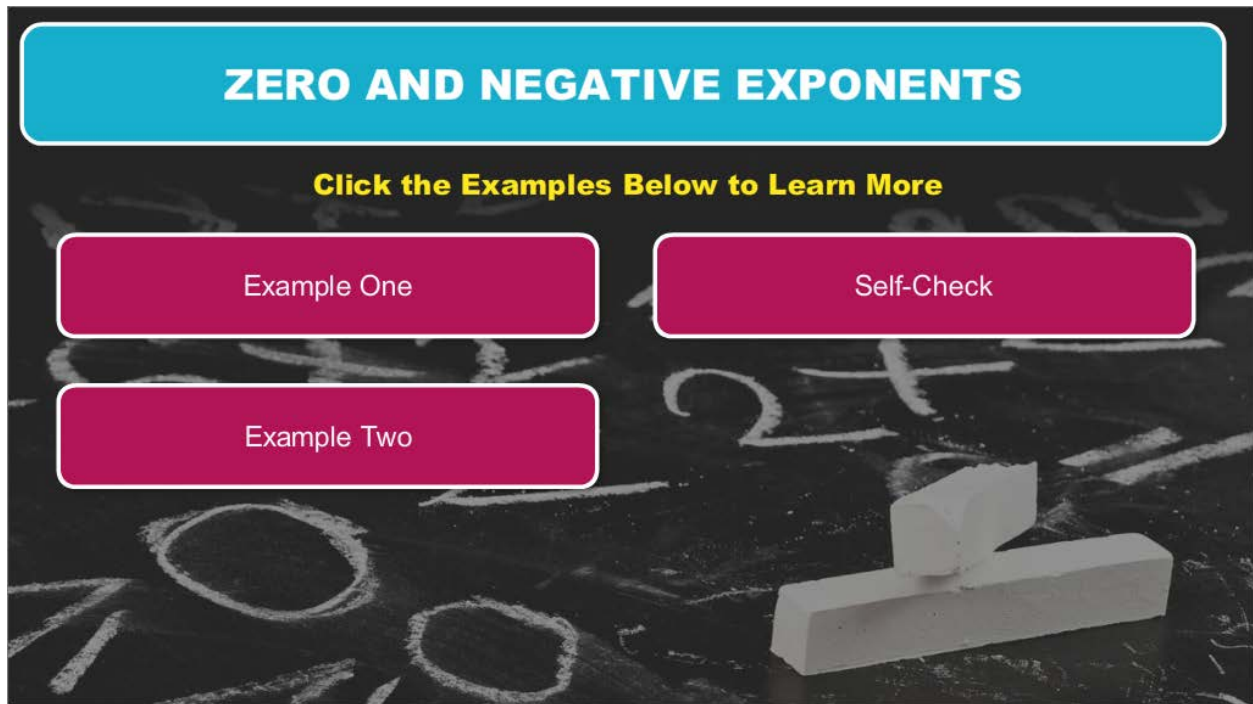
If $a \neq 0$,
then $a^{-r} = \frac{1}{a^r}$,
and $a^r = \frac{1}{a^{-r}}$

The definition of a negative exponent states that if $a \neq 0$, then $a^{-r} = \frac{1}{a^r}$, and $a^r = \frac{1}{a^{-r}}$.

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Zero and Negative Exponents

The image shows a digital interface for learning about zero and negative exponents. At the top, a blue rounded rectangle contains the title 'ZERO AND NEGATIVE EXPONENTS' in white capital letters. Below this, a yellow text prompt 'Click the Examples Below to Learn More' is centered. Underneath the prompt are three magenta rounded rectangular buttons: 'Example One' on the left, 'Self-Check' on the right, and 'Example Two' below 'Example One'. The background of the interface is a dark chalkboard with faint white chalk markings of numbers and symbols, and a small white 3D geometric model of a cube on a rectangular base is visible in the lower right corner.

ZERO AND NEGATIVE EXPONENTS

Click the Examples Below to Learn More

Example One

Self-Check

Example Two

Click the examples below the learn more.

Module 2: Properties of Exponents

Topic 3: Zero and Negative Exponents

Example 1

EXAMPLE 1

Simplify the expression: $\left(\frac{3x^6}{y^{-5}}\right)^4$

$$\left(\frac{3x^6}{y^{-5}}\right)^4 = \frac{(3x^6)^4}{(y^{-5})^4}$$

Is the expression above true or false?

True

False

Please click on the correct answer.

Simplify the following expression:

$$\left(\frac{3x^6}{y^{-5}}\right)^4$$

In this example, you must raise the quotient of $3x^6$ and y^{-5} to the fourth power. According to the Power of a Quotient Property, you must raise the numerator and denominator to the fourth power. Therefore,

A) True

B) False

Is the expression above true or false? Please click on the correct answer.

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Example 1 (continued)

Next

EXAMPLE 1

Simplify the expression: $\left(\frac{3x^6}{y^{-5}}\right)^4$

$$\left(\frac{3x^6}{y^{-5}}\right)^4 = \frac{(3x^6)^4}{(y^{-5})^4}$$

Is the expression above true or false?

True

According to the Power of a Quotient Property, you must raise each factor to the fourth power. Therefore, the expression above holds true.

$$\left(\frac{3x^6}{y^{-5}}\right)^4 = \frac{(3x^6)^4}{(y^{-5})^4}$$

Feedback: According to the Power of a Quotient Property, you must raise each factor to the fourth power. Therefore the expression above holds true.

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Example 1 (continued)

EXAMPLE 1

Simplify the expression: $\left(\frac{3x^6}{y^{-5}}\right)^4$

$$\left(\frac{3x^6}{y^{-5}}\right)^4 = \frac{(3x^6)^4}{(y^{-5})^4} = \frac{?}{?}$$

Which of the following expressions is equivalent to $(3x^6)^4$?

$12x^{10}$

$81x^{24}$

$12x^{12}$

To simplify the expression in the numerator, $(3x^6)^4$, you must apply the Power of a Product Property.

Which of the following expressions is equivalent to $(3x^6)^4$?

A) $12x^{10}$

B) $81x^{24}$

C) $12x^{12}$

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Example 1 (continued)

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EXAMPLE 1

Simplify the expression: $\left(\frac{3x^6}{y^{-5}}\right)^4$

$$\left(\frac{3x^6}{y^{-5}}\right)^4 = \frac{(3x^6)^4}{(y^{-5})^4} = \frac{81x^{24}}{y^{-20}}$$

According to the Power of a Product Property, you must raise each factor to the fourth power.

$$\begin{aligned}(3x^6)^4 &= 3^4 \cdot (x^6)^4 \\ &= 81 \cdot x^{24} \\ &= 81x^{24}\end{aligned}$$

$81x^{24}$

Feedback: According to the Power of a Product Property, you must raise each factor to the fourth power.

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Example 1 (continued)

EXAMPLE 1

Simplify the expression: $\left(\frac{3x^6}{y^{-5}}\right)^4$

$$\left(\frac{3x^6}{y^{-5}}\right)^4 = \frac{(3x^6)^4}{(y^{-5})^4} = \frac{81x^{24}}{?}$$

Which of the following expressions is equivalent to $(y^{-5})^4$?

$$y^{-1}$$

$$y^{-9}$$

$$y^{-20}$$

To simplify the expression in the denominator, you must apply the Power of a Power Property.

Which of the following expression is equivalent to $(y^{-5})^4$?

A) y^{-1}

B) y^{-9}

C) y^{-20}

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Example 1 (continued)

Next

EXAMPLE 1

Simplify the expression: $\left(\frac{3x^6}{y^{-5}}\right)^4$

$$\left(\frac{3x^6}{y^{-5}}\right)^4 = \frac{(3x^6)^4}{(y^{-5})^4} = \frac{81x^{24}}{y^{-20}}$$

According to the Power of a Power Property, you must multiply the exponents.

$$(y^{-5})^4 = y^{-5 \cdot 4} = y^{-20}$$

$$y^{-20}$$

Feedback: According to the Power of a Power Property you must multiply the exponents.

$$(y^{-5})^4 = y^{-5 \cdot 4} = y^{-20}$$

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Example 1 (continued)

EXAMPLE 1

Simplify the expression: $\left(\frac{3x^6}{y^{-5}}\right)^4$

$$\begin{aligned}\left(\frac{3x^6}{y^{-5}}\right)^4 &= \frac{(3x^6)^4}{(y^{-5})^4} = \frac{81x^{24}}{y^{-20}} \\ &= \boxed{81x^{24}y^{20}}\end{aligned}$$

Definition of a Negative Exponent

$$\frac{1}{a^{-r}} = a^r$$

Now you must use the Definition of a Negative Exponent in order to eliminate the negative power in the denominator.

According to the Definition of a Negative Exponent, $\frac{1}{a^{-r}} = a^r$. So, $\frac{81x^{24}}{y^{-20}} = 81x^{24}y^{20}$.

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Example 1 (continued)

Next

EXAMPLE 1

Simplify the expression: $\left(\frac{3x^6}{y^{-5}}\right)^4$

$$\begin{aligned}\left(\frac{3x^6}{y^{-5}}\right)^4 &= \frac{(3x^6)^4}{(y^{-5})^4} = \frac{81x^{24}}{y^{-20}} \\ &= 81x^{24}y^{20}\end{aligned}$$

Definition of a Negative Exponent

If the negative power is in the denominator, you must move it to the numerator, and make the exponent positive.

One way to think about this is if the term that includes the negative power is in the denominator, you must move it to the numerator, and make the exponent positive.

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Example 2

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$
$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4)$$

Enter the correct answer below and click submit.

$$-4 \cdot 2 =$$

Submit

In this example you must find the product of exponential expressions that include coefficients and more than one base. Recall that the Commutative Property of Multiplication states that you can multiply values in any order. Apply that property in this example, by changing the order of multiplication, so that you multiply the coefficients first, then f^3 and f^{-3} , and finally b^{-6} and b^4 .

$$-4 \cdot 2 = ?$$

Enter the correct answer below and click submit.

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Example 2 (continued)

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$

$$\begin{aligned} -4f^3b^{-6} \cdot 2f^{-3}b^4 &= (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4) \\ &= -8 \end{aligned}$$

-4 times 2 is equal to -8.

$$-4 \cdot 2 = -8$$

Feedback: $-4 \cdot 2 = -8$

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Example 2 (continued)

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$

$$\begin{aligned} -4f^3b^{-6} \cdot 2f^{-3}b^4 &= (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4) \\ &= -8 \end{aligned}$$

Enter the correct exponent below and click submit.

$$f^3 \cdot f^{-3} = f^{\boxed{}}$$

Submit

In order to multiply f^3 and f^{-3} , you will need to add the exponents.

$$f^3 \cdot f^{-3} = f^?$$

Enter the correct exponent below and click submit.

Module 2: Properties of Exponents

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Example 2 (continued)

Next

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$

$$\begin{aligned} -4f^3b^{-6} \cdot 2f^{-3}b^4 &= (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4) \\ &= -8 \cdot f^0 \end{aligned}$$

f^3 times f^{-3} is equal to f^0 .

$$f^3 \cdot f^{-3} = f^{3+(-3)} = f^0$$

Feedback: $f^3 \cdot f^{-3} = f^{3+(-3)} = f^0$

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Example 2 (continued)

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$

$$\begin{aligned} -4f^3b^{-6} \cdot 2f^{-3}b^4 &= (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4) \\ &= -8 \cdot f^0 \end{aligned}$$

Enter the correct exponent below and click submit.

$$b^{-6} \cdot b^4 = b^{\boxed{}}$$

Submit

In order to multiply b^{-6} and b^4 , you will need to add the exponents.

$$b^{-6} \cdot b^4 = b^?$$

Enter the correct exponent below and click submit.

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Example 2 (continued)

Next

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$

$$\begin{aligned} -4f^3b^{-6} \cdot 2f^{-3}b^4 &= (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4) \\ &= -8 \cdot f^0 \cdot b^{-2} \end{aligned}$$

b^{-6} times b^4 is equal to b^{-2} .

$$b^{-6} \cdot b^4 = b^{-6+4} = b^{-2}$$

Feedback: $b^{-6} \cdot b^4 = b^{-6+4} = b^{-2}$

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Example 2 (continued)

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$

$$\begin{aligned} -4f^3b^{-6} \cdot 2f^{-3}b^4 &= (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4) \\ &= -8 \cdot f^0 \cdot b^{-2} \\ &= -8 \end{aligned}$$

According to the Definition of a Zero Exponent, $f^0 = ?$

1

0

f

Continue simplifying the exponential expressions.

According to the Definition of a Zero Exponent, $f^0 = ?$

A) 1

B) 0

C) f

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Example 2 (continued)

[Next](#)

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$
$$\begin{aligned} -4f^3b^{-6} \cdot 2f^{-3}b^4 &= (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4) \\ &= -8 \cdot f^0 \cdot b^{-2} \\ &= -8 \cdot 1 \end{aligned}$$

According to the Definition of a Zero Exponent, if $a \neq 0$, then $a^0 = 1$,
Therefore, $f^0 = 1$.

1

Feedback: According to the Definition of Zero Exponents, if $a \neq 0$, then $a^0 = 1$. Therefore, $f^0 = 1$.

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Example 2 (continued)

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4)$$

$$= -8 \cdot f^0 \cdot b^{-2}$$

$$= -8 \cdot 1$$

According to the Definition of a Negative Exponent, $b^{-2} = ?$

$$b^2$$

$$\frac{1}{b^2}$$

$$0$$

You must use the Definition of a Negative Exponent in order to simplify b^{-2} . Recall that according to the Definition of Negative Exponents, $a^{-r} = \frac{1}{a^r}$.

So, $b^{-2} = ?$.

A) b^2

B) $\frac{1}{b^2}$

C) 0

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Example 2 (continued)

Next

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4)$$

$$= -8 \cdot f^0 \cdot b^{-2}$$

$$= -8 \cdot 1 \cdot \frac{1}{b^2}$$

According to the Definition of a Negative Exponent, if $a \neq 0$, then $a^{-r} = \frac{1}{a^r}$.

$$\text{Therefore, } b^{-2} = \frac{1}{b^2}.$$

$$\frac{1}{b^2}$$

Feedback: According to the Definition of a Negative Exponent, if $a \neq 0$, then $a^{-r} = \frac{1}{a^r}$.

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Example 2 (continued)

Next

EXAMPLE 2

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = ?$$

$$-4f^3b^{-6} \cdot 2f^{-3}b^4 = (-4 \cdot 2) \cdot (f^3 \cdot f^{-3}) \cdot (b^{-6} \cdot b^4)$$

$$= -8 \cdot f^0 \cdot b^{-2}$$

$$= -8 \cdot 1 \cdot \frac{1}{b^2}$$

$$= \boxed{\frac{-8}{b^2}}$$

Now simplify the product.

$$-8 \cdot 1 \cdot \frac{1}{b^2} = \frac{-8}{b^2}$$

Your work is complete.

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Self-Check

 **Self-Check**

Choose the expression that is equivalent to $\frac{d^{10}e^4}{d^{10}e^7}$

- ☐ de^3
- ☐ $\frac{d}{e^3}$
- ☐ $\frac{1}{e^3}$

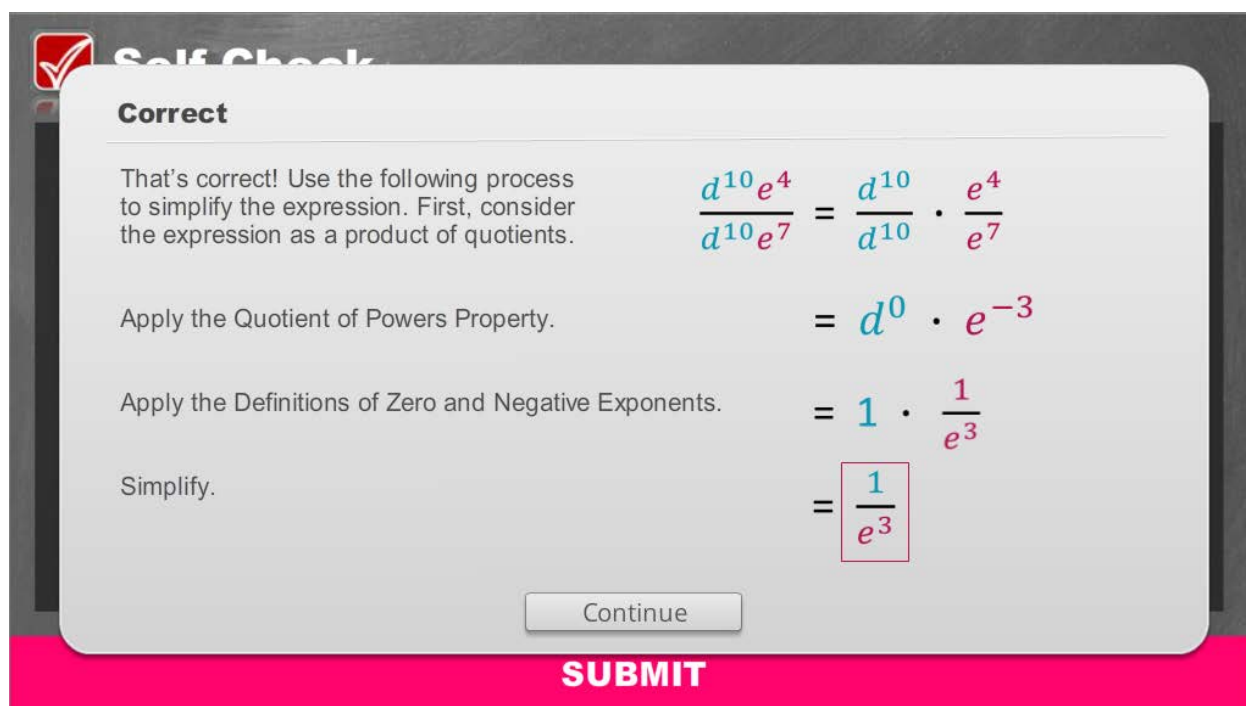
SUBMIT

Solve the problem in the image above to check your understanding of the content.

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Self-Check: Answer



Correct

That's correct! Use the following process to simplify the expression. First, consider the expression as a product of quotients.

$$\frac{d^{10}e^4}{d^{10}e^7} = \frac{d^{10}}{d^{10}} \cdot \frac{e^4}{e^7}$$

Apply the Quotient of Powers Property.

$$= d^0 \cdot e^{-3}$$

Apply the Definitions of Zero and Negative Exponents.

$$= 1 \cdot \frac{1}{e^3}$$

Simplify.

$$= \boxed{\frac{1}{e^3}}$$

Continue

SUBMIT

For your reference, the image above shows the correct solution to the self-check problem.

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Conclusion



The image shows a digital interface for a lesson conclusion. On the left, a white box with a pink header titled "Today's Lesson" contains two bullet points, each preceded by a checkmark. The first bullet point states: "✓ Learned the Definition of a Zero Exponent and raised a value to the power of zero". The second bullet point states: "✓ Learned the Definition of a Negative Exponent and raised a value to a negative power". Below the text are two pink buttons: "Exit Lesson" and "Restart Lesson". To the right of the box is a cartoon illustration of a young woman with dark skin, curly brown hair, and a pink long-sleeved shirt. The background is a solid blue color with a faint, repeating pattern of mathematical symbols like pi, infinity, and numbers.

Today's Lesson

- ✓ Learned the Definition of a Zero Exponent and raised a value to the power of zero
- ✓ Learned the Definition of a Negative Exponent and raised a value to a negative power

Exit Lesson

Restart Lesson

Congratulations! You have reached the conclusion of this lesson in Algebra I. In this lesson you applied your knowledge of exponents to discover definitions that allow you to raise a value to the power of zero or to a negative power.